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5. Differentiate the following with respect to x

$$\frac{e^{4x} \sin x}{x \cos 2x}$$

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Let $y = \frac{e^{4x} \sin x}{x \cos 2x}$

Let $u = e^{4x} \sin x$
 From u , let $t = e^{4x}$, $\frac{dt}{dx} = 4e^{4x}$
 $s = \sin x$, $\frac{ds}{dx} = \cos x$

$$\frac{du}{dx} = u \frac{dt}{dx} + t \frac{ds}{dx}$$

$$\begin{aligned}\frac{du}{dx} &= \sin x (4e^{4x}) + e^{4x} (\cos x) \\ &= 4e^{4x} \sin x + e^{4x} \cos x\end{aligned}$$

$$\frac{du}{dx} = e^{4x} (4 \sin x + \cos x)$$

Let $v = x \cos 2x$
 From v , let $P = x$, $\frac{dP}{dx} = 1$
 $w = \cos 2x$, $\frac{dw}{dx} = -2 \sin 2x$

$$\begin{aligned}\frac{dv}{dx} &= w \frac{dP}{dx} + P \frac{dw}{dx} \\ &= \cos 2x (1) + x (-2 \sin 2x) \\ &= \cos 2x + -2x \sin 2x\end{aligned}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{x \cos 2x [e^{4x} (4 \sin x + \cos x)] - e^{4x} \sin x (\cos 2x - 2x \sin 2x)}{(x \cos 2x)^2}$$

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$$\frac{dy}{dx} = \frac{x \cos 2x (4e^{4x} \sin x + \cos x) - e^{4x} \sin x (4\cos 2x - 2x \sin 2x)}{x^2 \cos^2 4x^2}$$

$$\frac{dy}{dx} = \frac{-4x e^{4x} \sin x \cos 2x + x \cos^2 2x^2 - e^{4x} \sin 2x \cos 2x + 2x e^{4x} \sin x}{x^2 \cos^2 4x^2}$$

$$\frac{dy}{dx} = \frac{e^{4x} \sin x \cos 2x (4x - 1) + x \cos^2 2x^2 + 2x e^{4x} \sin 2x}{x^2 \cos^2 4x^2}$$

$$\frac{dy}{dx} = \frac{e^{4x} \sin x \cos 2x (4x - 1) + x (\cos^2 2x^2 + x e^{4x} \sin 2x)}{x^2 \cos^2 4x^2}$$

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$$G_1 \quad \frac{x^4}{(x+1)^2}$$

$$\text{Let } u = x^4, \frac{du}{dx} = 4x^3 \\ v = (x+1)^2, \frac{dv}{dx} = 2 \cdot (x+1)^{2-1} \cdot 1 = 2(x+1) = 2x+2.$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{(x+1)^2 (4x^3) - x^4 (2x+2)}{(x+1)^4}$$

$$= \frac{4x^3 (x+1)^2 - x^4 (2x+2)}{(x+1)^4}$$

$$= \frac{12x^3 (x^2+2x+1) - 2x^5 - 2x^4}{(x+1)^4}$$

$$= \frac{4x^5 + 8x^3 + 4x^3 - 2x^5 - 2x^4}{(x+1)^4}$$

$$= \frac{4x^5 - 2x^5 - 2x^4 + 12x^3}{(x+1)^4}$$

$$= \frac{2x^5 - 2x^4 + 12x^3}{(x+1)^4} \quad \text{OR} \quad \frac{2x^5 - 2x^4 + 12x^3}{3x^4 + 2x^3 + 6x^2 + 4x + 1}$$

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$$6. y = \ln(3 - 4 \cos x)$$

Sol

$$y = \ln(3 - 4 \cos x)$$

$$\frac{dy}{dx} = \frac{1}{3 - 4 \cos x} \cdot (-4 \sin x)$$

$$y = \ln u$$

$$\frac{dy}{du} = \frac{1}{u} \cdot u'$$

$$\frac{du}{dx} = \frac{1}{3 - 4 \cos x} \cdot (3 + 4 \cos x)$$

$$\frac{dy}{dx} = \frac{4 \sin x}{3 - 4 \cos x}$$

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$$2. If x^2 + y^3 - 4x + 4y = 26 \text{ find } \frac{dy}{dx} \text{ and } \frac{d^2y}{dx^2}$$

Sol

$$x^2 + y^3 - 4x + 4y = 26$$

$$2x + 3y^2 \frac{dy}{dx} - 4 + 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \frac{3y^2 + 4}{3y^2 + 4} = 4 - 2x$$

$$\frac{dy}{dx} = \frac{4 - 2x}{3y^2 + 4}$$

To get $\frac{d^2y}{dx^2}$

$$\text{let } u = 4 - 2x$$

$$u = 3y^2 + 4$$

$$\frac{du}{dx} = -2$$

$$\frac{dv}{dx} = 6y \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{V \frac{du}{dx} - u \frac{dv}{dx}}{V^2}$$

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$$= \frac{(3y^2 + 4)(-2) - (4 - 2x)(6y \frac{dy}{dx})}{(3y^2 + 4)^2}$$

$$= \frac{(-6y^2 - 8) - (24y \frac{dy}{dx} - 12xy \frac{dy}{dx})}{(3y^2 + 4)^2}$$

$$= \frac{(-6y^2 - 8) - 24y \frac{dy}{dx} + 12xy \frac{dy}{dx}}{(3y^2 + 4)^2}$$

$$= \frac{-6y^2 - 8 - (24y - 12xy) \frac{dy}{dx}}{(3y^2 + 4)^2}$$

$$= \frac{-6y^2 - 8 - (24y - 12xy) \frac{(4 - 2x)}{(3y^2 + 4)}}{(3y^2 + 4)^2}$$

$$= -6y^2 - 8 - \left[\frac{96y - 48yx}{3y^2 + 4} - \frac{48xy^2 - 24x^2y}{3y^2 + 4} \right]$$

$$(3y^2 + 4)^2$$

$$-6y^2 - 8 - \left[\frac{96y - 48yx - 48x^2y + 24x^2y}{3y^2 + 4} \right] (3y^2 + 4)^2$$

$$\frac{d^2y}{dx^2} = \frac{-6y^2 - 8 - \cancel{96y + 48xy + 48x^2y} - 24x^2y}{\cancel{3y^2 + 4} (3y^2 + 4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{3y^2 + 4(-6y^2) - 8(3y^2 + 4) - \cancel{96y + 48xy + 48x^2y} - 24x^2y}{\cancel{3y^2 + 4} (3y^2 + 4)^2}$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{3y^2 + 4(-6y^2 - 8) - 96y + 96xy - 24x^2y}{3y^2 + 4} \times \frac{(3y^2 + 4)}{1} \\ &= (3y^2 + 4)^2 (-6y^2 - 8) - (96y + 96xy - 24x^2y)(3y^2 + 4)\end{aligned}$$

$$\frac{d^2y}{dx^2} = 3y^2 + 4 [3y^2 + 4 (-6y^2 - 8) - 96y - 96xy + 24x^2y]$$

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